METHODS FOR THE COMPUTATION OF TRIANGULATION ON THE GRID SYSTEM

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29TH ENGINEERS, U. S. ARMY
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A Resume of Lectures by Captain P. Ardon of the French Mission and 1st Lieut. Earl Church, 29th Engineers, U. S. Army, Supplemented with Results of Studies and Investigations by the Department of Computation and Triangulation, 1st Battalion, 29th Engineers.

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Methods for the Computation of Triangulation on the Grid System

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In a system of Map Projection in which points are located by rectangular coordinates, the solution of the triangles resulting from a survey may be greatly simplified by the use of certain methods of computation, which for the most part have been developed since the beginning of the War. The grid system as used in France has been fully described in a pamphlet on "Military Geodesy in France", and will not be discussed in this paper.

Y-AZIMUTH AND DISTANCE

Consider two points, A and B, joined by the line AB. The coordinates of A are x_A, y_A; and those of B are x_B, y_B. Now the Y-azimuth of the line AB is its inclination to the Y-line through A, measured clockwise around the circle from the north. It is generally called V_{AB}. It can be seen that the Y-azimuth of the line BA, or V_{BA}, is equal to $200^G + V_{AB}$.

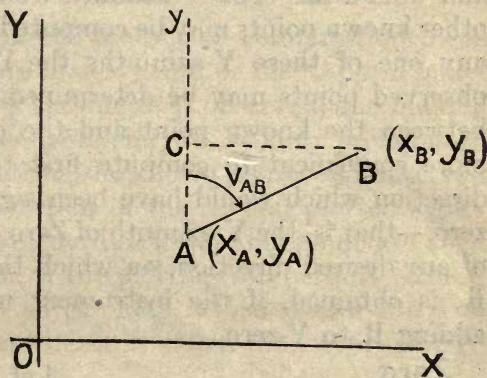


Figure I.

Let Ay be the Y-line through A, and drop the perpendicular BC from B to Ay. $BC = x_A - x_B$, and $AC = y_A - y_B$. In the right triangle ABC,

$$\tan CAB = BC/AC = (x_A - x_B)/(y_A - y_B).$$

If angle CAB is called v, we have $\tan v = \dfrac{(x_A - x_B)}{(y_A - y_B)}$ (Eq. 1)

When the differences $(x_A - x_B)$, $(y_A - y_B)$ are taken in magnitude without regard to sign, the angle v thus computed is the small angle by which line AB is inclined to the Y-line. The Y-azimuth is easily derived from angle v, if the respective positions of A and B are noted.

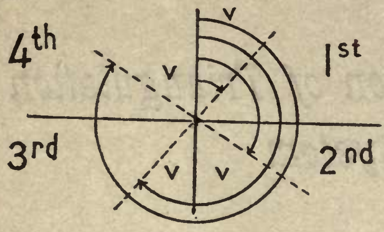


Figure 2.

If B is in the 1st quadrant, V=v.

" 2nd " $V=200^G - v$.

" 3rd " $V=200^G + v$.

" 4th " $V=400^G - v$.

In figure 1, let D equal the distance from A to B. Then,

$$D = AB = \frac{BC}{\sin v} = \frac{AC}{\cos v}$$

$$\text{or } D = \frac{x_A - x_B}{\sin v} = \frac{y_A - y_B}{\cos v} \qquad \text{(Eq. 2)}$$

Y-AZIMUTH OF THE ZERO

Suppose the instrument has been set up at a known point, and a set of readings has been taken on several other points, both known and unknown. The Y-azimuth from the instrument to each of the other known points may be computed, by means of Equation 1. From any one of these Y-azimuths the Y-azimuth to one of the unknown observed points may be determined by means of the angle observed between the known point and the unknown point. It is, however, more convenient to compute first the Y-azimuth of the imaginary direction which would have been sighted if the instrument had read zero,—that is, the Y-azimuth of Zero, or V-zero. Then the Y-azimuth of any desired direction, on which the reading of the instrument was R, is obtained, if the instrument used is graduated clockwise, by adding R to V-zero.

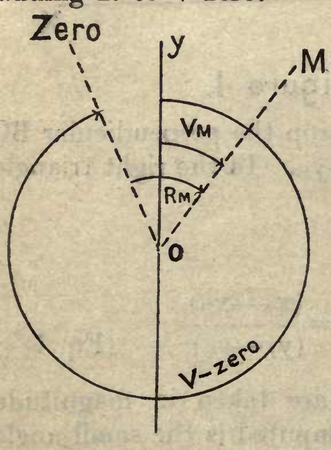


Figure 3.

Let x,y be the coordinates of the point occupied, and x_M, y_M x_N, y_N the coordinates of the known sighted points.

$$\tan v_M = (x_M - x) / (y_M - y)$$
$$\tan v_N = (x_N - x) / (y_N - y)$$

From v_M and v_N we determine the Y-azimuths to the points M and N, namely V_M and V_N. Let R_M and R_N be the corresponding readings on the instrument.

V_M=angle yOM

V-zero=angle y-O-zero.

R_M=angle zero-O-M.

Then the angle y-O-zero$=$yOM$+$M-O-zero
$=V_M+400^G-R_M$.

Or, $\quad$ V-zero$=V_M-R_M$.
Similarly, V-zero$=V_N-R_N$.

If there has been no error in observing or reading angles, the differences between the values of V-zero may show the degree of reliability of the sighted points. When the operation under consideration concerns local tertiary triangulation, the arithmetic mean is adopted. If more accuracy is desired, we can give the different observations weights which are proportional to the distances to the observed points, since angular error caused by linear error in sighting the points is inversly proportional to their distances. Thus, let V-zero$_A$ and V-zero$_B$ be the values of V-zero deduced from V_A and V_B. And let D_A and D_B, the distances to points A and B, be to each other as 3:4. Then, instead of adopting (V-zero$_A$+V-zero$_B$)/2 as the value of V-zero, we should adopt (3 V-zero$_A$+4 V-zero$_B$)/7.

LOCATION OF A POINT BY INTERSECTION METHOD

The principal characteristic of the method of computation to be described here is the use of an approximate point, or the "Approximate Point Method." Instead of taking the angles as observed, and making a direct analytical solution of the problem, we make a preliminary graphical solution, by actually plotting the known points and observed angles to a small scale, the intersection of the several plotted sights locating the unknown point. The coordinates of this point are then scaled off as accurately as the scale of the plotting will permit. These coordinates define a certain point on the map, which is near the true position of the unknown point, and hence is called the "approximate point".

The next step is to compute the corrections to apply to the coordinates of the approximate point in order to obtain the true coordinates of the desired point. Let us call A the approximate point and O the desired point. In Figure 4, let M be one of the known points from which O has been sighted. The reading on O is equal to R. The V-zero at M having already been determined, we can calculate the Y-azimuth of the *observed* direction, MO, by the formula.

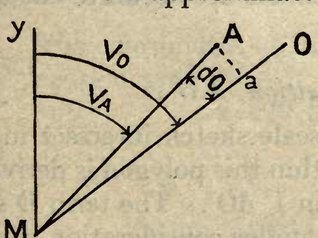


Figure 4.

$$V_O = \text{V-zero} + R.$$

This angle is equal to angle yMO in Figure 4. The Y-azimuth from M to the approximate point A may be calculated, as explained on P.5 (Eq.1), since the coordinates of both M and A are *known*. This gives us V_A, which is angle yMA.

Now from A drop a perpendicular, Aa, to the line MO. The angle AMa, or dO, is equal to angle yMO minus angle yMA, or to $V_O - V_A$. Then we have, in the triangle AMa, $Aa = MA \cdot \sin dO$.
The distance MA, or D, may be calculated from the coordinates of M and A, as shown on P. 6 (Eq. 2). If dO is less than three grades, we can replace sin dO by dO(in minutes)xsin 1′, or dO′ sin 1′.

Hence, $Aa = DdO' \sin 1'$. (Eq. 3)

Now, since Aa has been computed and the direction V_O is known, we can plot on a large scale sketch the locus containing O as determined by the observation at M. Such a locus can be drawn for each intersection observation taken on the desired point from the other known points. The common intersection of these loci gives the desired point O. From the coordinates of A the coordinates of O may be determined by scaling from the large scale sketch the differences in their X and Y coordinates. The intersections of the above loci, however, usually give a triangle or polygon of error, due to discrepancies in the original observed angles, or in the coordinates of the known points. The selection of the desired point within this polygon of error will be discussed below.

Remarks.—(1) The above reasoning is independent of the distance between A and O. It is, however, necessary to construct point A as accurately as possible so as to draw the sketch showing O to a large scale.

(2) In field triangulation, the instruments used allow angular measurements within 1′ or 2′. Five decimals are therefore sufficient in computing v_A by logarithms. If the construction has been carefully made, Aa is smaller than 10^m, and four decimals are accurate enough for this part of the computation.

To Select the Most Probable Position of O

If the loci when plotted on the large scale sketch intersect in a polygon, the most probable position of O within this polygon is derived from the examination of the quantities D sin 1′ dO′. The term D sin 1′ represents the displacement of the locus under consideration when the Y-azimuth changes by 1′. Hence the relative "sensitivities" of the loci may be considered as being proportional to the distances from the known points to the desired point.

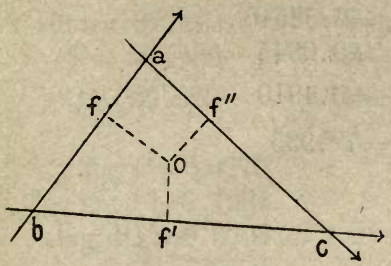


Figure 5.

Suppose for instance, in Figure 5, that the triangle of error abc is obtained. The problem is to locate O in such a manner that the perpendicular distances to the loci will be respectively proportional to the sensitivities of the loci; or so that

$$of/D = of'/D' = of''/D'',$$

where D, D' and D'' are distances from A to the known points, from which observations were taken. The graphical location of O may be made by replacing the triangle of error by a smaller similar triangle within, its sides distant from the original loci by amounts respectively proportional to D, D' and D''.

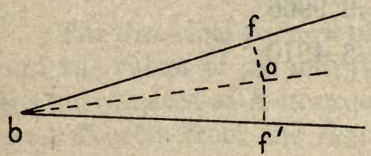


Figure 6.

Another method is to replace two loci by a single line, such that

$$of/D = of'/D'. \qquad \text{(Figure 6.)}$$

EXAMPLE OF INTERSECTION METHOD

The formulas are $\tan v = (x_M - x_A)/(y_M - y_A)$ (Eq. 1)

$D = (x_M - x_A)/\sin v_M = (y_M - y_A)/\cos v_M$ (Eq. 2)

$Aa = q = D \sin 1' \, dO'$. (Eq. 3)

From stations S_1, S_2 and S_3 point O has been sighted.

Data. *Sight from*	*X*	*Y*	*V-zero*	*R*
S_1	29 010	441 344	$311^G.181$	$96^G.775$
S_2	32 423	442 474	297 .457	38 .837
S_3	26 608	440 566	8 .440	31 .480

The coordinates of the approximate point A derived from graphcal location are:

$x = 29\ 395 \qquad y = 444\ 417$

Computation

S_1.

$x_S = 29\ 010 \qquad y_S = 441\ 344$

$x_A = 29\ 395 \qquad y_A = 444\ 417$

$(x_S - x_A) = 385 \qquad (y_S - y_A) = 3\ 073$

$\log(x_S - x_A) = 2.58546$ $\log(x_S - x_A) = 2.58546$
$\log(y_S - y_A) = 3.48756$ $\log \sin v = 9.0945$
$\log \tan v = 9.09790$ $\log D = 3.4910$

$\log D = 3.4910$ $v = 7^G.935$

V-zero $= 311^G.181$ $V = 7^G.935$
$R = 96.775$ $O = 7.956$
$O = 7.956$ $dO = 2'.1$

$\log D = 3.4910$
$\log \sin 1' = 6.1961$ $\log(y_S - y_A) = 3.4876$
$\log dO' = 0.3222$ $\log \cos v = 9.9966$
$\log q = 0.0093$ $\log D = 3.4910$
$q = 1^m.0$

S2.

$x_S = 32\ 423$ $y_S = 442\ 474$
$x_A = 29\ 395$ $y_A = 444\ 417$
$(x_S - x_A) = 3\ 028$ $(y_S - y_A) = 1\ 943$

$\log(x_S - x_A) = 3.48116$ $\log(x_S - x_A) = 3.4812$
$\log(y_S - y_A) = 3.28847$ $\log \sin v = 9.9251$
$\log \tan v = 0.19269$ $\log D = 3.5561$

$\log D = 3.5561$ $v = 63^G.681$

V-zero $= 297^G.457$ $V = 336^G.319$
$R = 38.837$ $O = 336.294$
$O = 336.294$ $dO = 2'.5$

$\log D = 3.5561$
$\log \sin 1' = 6.1961$ $\log(y_S - y_A) = 3.2885$
$\log dO' = 0.3979$ $\log \cos v = 9.7324$
$\log q = 0.1501$ $\log D = 3.5561$

$q = 1^m.4$

S3.

$x_S = 26\ 608$ $y_S = 440\ 566$
$x_A = 29\ 395$ $y_A = 444\ 417$
$(x_S - x_A) = 2\ 787$ $(y_S - y_A) = 3\ 851$

$\log(x_S - x_A) = 3.44514$ $\log(x_S - x_A) = 3.4451$
$\log(y_S - y_A) = 3.58557$ $\log \sin v = 9.7681$
$\log \tan v = 9.85957$ $\log D = 3.6770$

$\log D = 3.6770$ $v = 39^G.882$

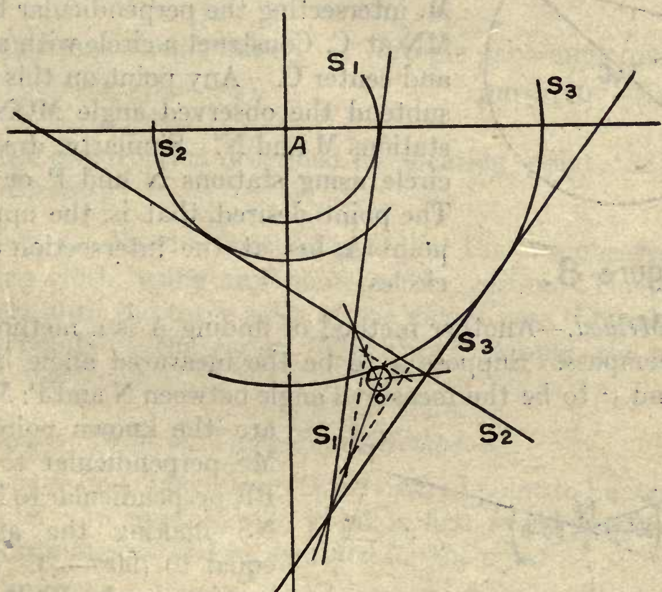
V-zero=	$8^G.440$	V=	$39^G.882$
R=	31.480	O=	39.920
O=	39.920	dO′=	3′.8

log D=3.6770
log sin 1′=6.1961
log dO′=0.5798
log q=0.4529

q=$2^m.8$

log $(y_S - y_A)$=3.5856
log cos v=9.9086
log D=3.6770

The three loci are now plotted to a large scale as shown in Figure 7, and the values of Δx and Δy, which are the quantities to be added to the coordinates of A in order to find the coordinates of O, are measured directly from the sketch.



Scale:– 1 cm = 0.8 m

Figure 7.

x_A=29 395.0	y_A=444 417.0
Δx= + 1.0	Δy= — 2.7
x_O=29 396.0	y_O=444 414.3

LOCATION OF A POINT BY THE THREE-POINT METHOD

Determination of the approximate point A.—

If from the desired point observations are taken on known points, M, N and P, the point is located by the Three-Point Method. In this method, as in the intersection problem, we make use of an "approximate point", A, in order to determine the true coordinates of the desired point, O. We now consider the graphical methods of determining the approximate point.

1st Method.—Two circles can be plotted; one being the locus of all points which will subtend the observed angle MON, and the other the locus of all points which will subtend the observed angle NOP. Each circle may be drawn as follows: at M, lay off the observed angle MON, as the angle α. To the line thus determined, draw a perpendicular at M, intersecting the perpendicular bisector of MN at C. Construct a circle with radius MC, and center C. Any point on this circle will subtend the observed angle MON between stations M and N. Similarly, draw another circle using stations N and P or M and P. The point desired, that is, the approximate point A, lies at the intersection o the two circles.

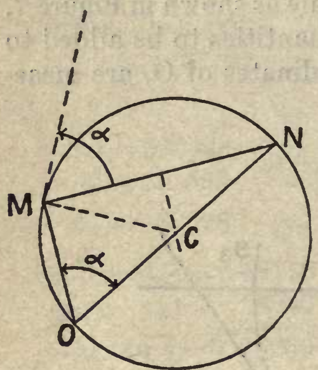


Figure 8.

2nd Method.—Another method of finding A is a method without using a compass. Suppose α to be the measured angle between M and N, and β to be the measured angle between N and P; M, N and P are the known points. Draw MS perpendicular to MN, and PR perpendicular to NP. Draw NS making the angle MNS equal to $(90^{\circ}-\alpha)$. Draw NR making angle PNR equal to $(90^{\circ}-\beta)$. These two lines intersect MS and PR in S and R. Draw RS. Draw NA, from N perpendicular to RS. Then A represents the approximate point.

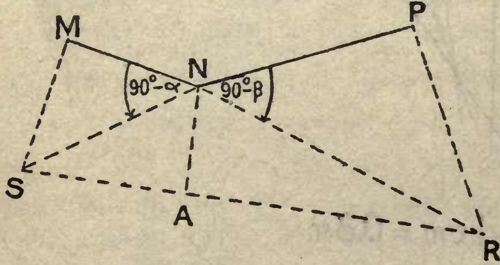


Figure 9.

Proof.—Suppose m, n and p to be plotted from known points, and suppose a, the approximate point, to be already located. Suppose circles mna and npa to be drawn. Draw ms perpendicular to mn, cutting the circle at s, and draw pr perpendicular to np cutting the other circle at r. Draw as and ar. Since smn and npr are right angles, ns and nr are diameters. Hence angles nas and nar are right angles, and sar is a straight line,

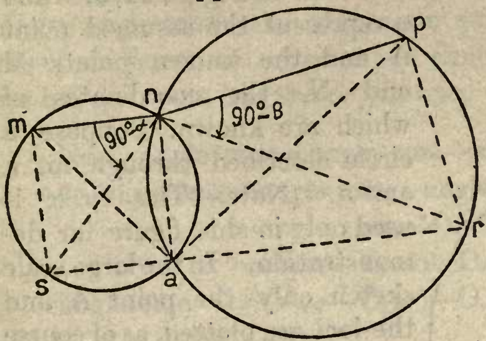


Figure 10.

Now angle man $=\alpha$ nap $=\beta$

" mas $=90^\circ-\alpha$ par $=90^\circ-\beta$

But angle mas = angle mns. Angle par = angle pnr

Hence, " mns $=(90^\circ-\alpha)$. " pnr $=(90^\circ-\beta)$.

Therefore the construction described for locating point "a" is geometrically correct.

3rd Method.—Another method is to plot the two observed angles upon tracing cloth, using any point as the vertex. Then shift the tracing cloth until the three sides of the angles pass throught he corresponding points on the paper. Then prick through the paper the position of "a."

Determination of point by Three-Point Method.—

Sights have been taken from O the desired point to known stations M, N, P, etc. An approximate point is first found graphically and definite coordinates x_A and y_A assumed for the point A.

Then next we consider the observations in pairs, as for instance the two known points M and N. The problem is to draw a locus containing O by means of a distance computed from A. This locus is drawn on a large scale figure. The same operation is performed with the observations on N and P, giving another locus which intersects the first one in the desired point. Then the values of Δx and Δy or the amounts to be added to x_A and y_A to obtain the coordinates of O, are scaled from this large scale drawing. In case a check is made, or several three-point problems observed, a point O is chosen from the triangle or polygon of error as explained later.

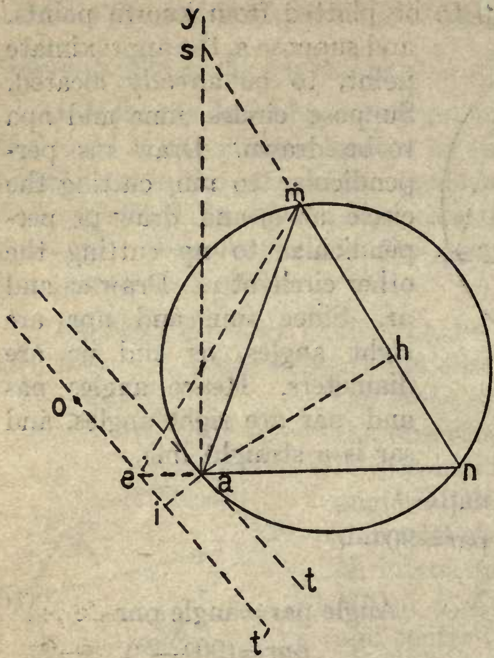


Figure 11.

Now in Figure 11, a, m and n represent the assumed point A and the known points M and N, the coordinates of which are known. Suppose a circle described through m, n and a. (Note.—This circle is used only in this figure for demonstration. In the large scale sketch only the point A and the loci are plotted, as of course the points M and N would be off the sketch).

Through a, draw a tangent at. Suppose o to be the location of the desired point. The problem now is to compute the displacement ai of the circle, so that the tangent or the circle (they coincide in the large scale sketch) will pass through o. Also, as points a and o are close together, tangent t′o to circle nmo may be considered as parallel to at.

Draw ah perpendicular to mn, and ai perpendicular to at and ot′. Prolong na to meet ot′ in e, and draw em. Now, in the triangles amh and aei, the angle aei (=angle nat) is measured by arc na /2 as is also angle amh. And the triangles are right triangles. Therefore they are similar. Hence,

ai /ah = ae /am or ai = ah-ae /am.
or q, the displacement desired, equals ah-ae /am.

Now let us consider the triangle aem.
am /sin aem = ae /sin ame. (Eq. 4)

But we have assumed that the circle mon coincides with the tangent in the vicinity of o, when the sketch is to a large scale. Therefore the angle men is equal to angle mon, being measured by one-half the same arc in the same circle.

Angle men = angle mon = R_N—R_M; where R_M and R_N are the readings of the instrument at O, on M and N respectively.

Angle men=R_N—R_M. Call R_N—R_M the observed angle, O.
Then, angle men=O. Hence sin aem=sin O.
Now, since am/sin aem=ae/sin ame by Equation (4),
we have, am/sin O=ae/sin ema (Eq. 5)

Now, angle man=angle ema+angle mea.
or angle ema=angle man—angle mea
=angle man—O.
=(Yan—Yam)—O.
angle ema =(V_{AN}—V_{AM})—O.

V_{AN} and V_{AM}, the Y-azimuths from A to N and M respectively, are easily computed since the coordinates of A, N and M are known, from the formula:

$$\tan V_{AN} = \frac{x_N - x_A}{y_N - y_A}$$

$$\tan V_{AM} = \frac{x_M - x_A}{y_M - y_A}$$

Call the angle ema, dO. That is, dO=(V_{AN}—V_{AM})—O.
In words, dO is the difference between the two values: (1) the difference between the computed Y-azimuths to M and N from A; and (2) the observed angle at O between M and N.

Now, q=ah-ae/am (from above)
or calling ah=h; q=h-ae/am.
But from Eq. 5, above ae/am=sin ema/sin O.
Hence, q=h-sin ema/sin O.
q=h-sin dO/sin O.

Or, when dO is small,

$$q = \frac{h}{\sin O} \cdot \sin 1' \, dO \text{ (in minutes).} \quad \text{(Eq. 6)}$$

Next, let us work out another formula for h.
am/sin mna=mn/sin man.
Call am=d; an=d′; mn=D.
Then, d/sin mna=D/sin man.
Or, d/D =sin mna/sin man. Multiply by d′.
Then, dd′/D=d′ sin mna/sin man.
Or, dd′/D=h/sin man. (Eq. 7)

Now, from above, angle man=angle ema+O
$=dO+O$

Hence, sin *man*=sin dO cos O+cos dO sin O.
But, when dO is very small, sin dO=zero; and cos dO=1 (approximately).
Hence, sin man=sin O.
Substituting this in Equation 7 above gives

$$\frac{dd'}{D} = \frac{h}{\sin O} \qquad \text{(Eq. 8)}$$

Substitute Equation 8 in Equation 6, and we have,

$$q=\frac{dd'}{D} \sin 1' \, dO'. \qquad \text{(Eq. 9)}$$

This formula gives the required value of the displacement of the circle; and since the circle and its tangent coincide near O for the large scale sketch, the locus containing the required point is a line whose distance from A is given by the above formula for q.

Values d, d' and D are easily found from the coordinates of M, N and A; and dO, equal to $(V_{AN}-V_{AM})-O$, is easily found since O is measured and V_{AN} and V_{AM} can easily be found from the coordinates of M, N and A.

The distance from A to the line containing O is, then, computed. It remains to find its direction. It should be remembered that since o and a are close together, it was stated that we can consider at and ot' parallel.

Angle *Yat=Yan+nat=Van+nat=Van+amn.*

Produce nm to meet aY in s. Then, *amn=asm+sam=asm+Vam.*

Hence, Angle Yat $=V_{AN}+V_{AM}+$angle asm
$=V_{AN}+V_{AM}+(180^{o}-V_{MN})$
$=V_{AN}+V_{AM}-V_{MN}$

Calling angle Yat, the Y-azimuth of the tangent "at", V_S, we have

$$V_S=V_{AM}+V_{AN}-V_{MN} \qquad \text{(Eq. 10)}$$

The three Y-azimuths V_{AM}, V_{AN} and V_{MN} may easily be computed from the coordinates of M, N and A.

The following two formulas, then, by giving the distance q of the tangent from A, and the direction V_S of the tangent, locate this locus containing O:

$$q = \frac{dd'}{D} \sin 1' \, dO' \qquad \text{(Eq. 9)}$$

$$V_S = V_{AM} + V_{AN} - V_{MN} \qquad \text{(Eq. 10)}$$

Remarks.—(1) When the point O is determined by the intersection of several of these lines, the point is finally chosen from the triangle or polygon of error. This is done remembering that since

$$q = \frac{dd'}{D} \sin 1' \, dO',$$

the sensitivity of any of these lines is

$$\frac{dd'}{D} \sin 1',$$

which is the change in q for a change of 1′ in dO. The sensitivity and weight are reciprocal values. Obviously, the sensitivities of loci are directly proportional to their respective values for dd′/D.

(2) Three known points locate the desired point. Four give one check.

(3) To check the computations, if there are three points only, three lines may be obtained by choosing three of the different pairs of known points. For instance, if the known points are M, N and P, by taking the angle on M and N one locus is obtained; on M and P another; on N and P a third. *If the work is correct these three lines should intersect in a point,* for the three points M, N and P are only sufficient to locate the desired point without check.

(4) Measurements of d, d′ and D on the board are sufficiently accurate, if graphical drawing has been carefully carried on, that is if A is close to point O. It is to be remembered that four decimal places are sufficient in the logarithms when computing q, and that, consequently, a great accuracy is not essential in these data. Their computation is only necessary when points to be located are points of primary triangulation, or when some of the points used are not contained on the board.

(5) The position of the locus relative to the approximate point (that is, the side of the circle of radius q on which the locus is plotted) must be determined by examination of the plotted positions of the known points. Determine by inspection the location of the center of the circle through the two known points, M and N, and the approxi-

mate point A. Let O be the angle observed at the true point, between M and N; and let C be the computed angle at A between M and N. ($C = V_{AM} - V_{AN}$)

Then if C is greater than O, the locus will lie on the side of A *away from* the center. If C is smaller than O, the locus will lie on the side of A *towards* the center.

EXAMPLE OF THREE-POINT METHOD

From the desired point O, sights have been taken and readings noted as follows:

Sight on	*R*	*X*	*Y*
M	$50^G.39$	44 875	461 187
N	106 .50	49 989	464 141
P	365 .10	51 507	459 580
Q	39 .69	48 688	462 438

Coordinates of Approx. Pt. A:
X=50 707
Y=463 785

Computations

M.

$x_M = 44\ 875$ $\quad y_M = 461\ 187$ $\quad \log(x_M - x_A) = 3.76582$

$x_A = 50\ 707$ $\quad y_A = 463\ 785$ $\quad \log(y_M - y_A) = 3.41464$

$(x_M - x_A) = 5\ 832$ $\quad (y_M - y_A) = 2\ 598$ $\quad \log\tan v_M = 0.35118$

$v_M = 73^G.320$

$V_M = 273.320$

$$\log(x_M - x_A) = 3.7658$$
$$\log\sin v_M = 9.9607$$
$$\log D = 3.8051$$
$$\log(y_M - y_A) = 3.4146$$
$$\log\cos v_M = 9.6095$$
$$\log D = 3.8051$$

N.

$x_N = 49\ 989$ $\quad y_N = 464\ 141$ $\quad \log(x_N - x_A) = 2.85612$

$x_A = 50\ 707$ $\quad y_A = 463\ 785$ $\quad \log(y_N - y_A) = 2.55145$

$(x_N - x_A) = 718$ $\quad (y_N - y_A) = 356$ $\quad \log\tan v_N = 0.30467$

$v_N = 70.696$

$V_N = 329.304$

$$\log(x_N - x_A) = 2.8561$$
$$\log\sin v_N = 9.9523$$
$$\log D = 2.9038$$
$$\log(y_N - y_A) = 2.5515$$
$$\log\cos v_N = 9.6477$$
$$\log D = 2.9038$$

P.

$x_P = 51\ 507$	$y_P = 459\ 580$	$\log (x_P - x_A) = 2.90309$
$x_A = 50\ 707$	$y_A = 463\ 785$	$\log (y_P - y_A) = 3.62377$
$(x_P - x_A) = 800$	$(y_P - y_A) = 4\ 205$	$\log \tan v_P = 2.27932$
		$v_P = 11.969$
		$V_P = 188.031$

$\log (x_P - x_A) = 2.9031$
$\log \sin v_P = 9.2716$
$\log D = 3.6315$
$\log (y_P - y_A) = 3.6238$
$\log \cos v_P = 9.9923$
$\log D = 3.6315$

Q.

$x_Q = 48\ 688$	$y_Q = 462\ 438$	$\log (x_Q - x_A) = 3.30514$
$x_A = 50\ 707$	$y_A = 463\ 785$	$\log (y_Q - y_A) = 3.12937$
$(x_Q - x_A) = 2\ 019$	$(y_Q - y_A) = 1\ 347$	$\log \tan v_Q = 0.17577$
		$v_Q = 62.545$
		$V_Q = 262.545$

$\log (x_Q - x_A) = 3.3051$
$\log \sin v_Q = 9.9201$
$\log D = 3.3850$
$\log (y_Q - y_A) = 3.1294$
$\log \cos v_Q = 9.7444$
$\log D = 3.3850$

NOTE.—In the remainder of the computations, the values of D in the formula

$$\frac{dd'}{D} \sin 1' \, dO'$$

were scaled from the board. The computation of V_{MQ}, V_{PQ} and V_{NQ} is not given.

1.—*Q-M*

$R_Q = 39.69$	$V_Q = 262.545$	$\log d = 3.3850$
$R_M = 50.39$	$V_M = 273.320$	$\log d' = 3.8051$
$O = 10.70$	$C = 10.775$	$\log \sin 1' = 6.1961$
$C = 10.775$	$(V_Q + V_M) = 135.865$	$\text{colog } D = 6.3965$
$dO = 7'.5$	$V_{MQ} = 79.8$	$\log \frac{dd' \sin 1'}{D} = 9.7827$ $S = 0.6$
	$V_S = 56.0$	
		$\log dO = 0.8751$
		$\log q = 0.6578$ $q = 4.50$

2.—*Q-P*

R_Q = 39.69	V_Q = 262.545	log d = 3.3850			
R_P = 365.10	V_P = 188.031	log d′ = 3.6315			
O = 74.59	C = 74.514	log sin 1′ = 6.1961			
C = 74.514	(V_Q+V_P) = 450.5	colog D = 6.3964			
dO = 7′.6	V_{PQ} = 150.4	$\log \frac{dd' \sin 1'}{D}$ = 9.6090	S = 0.4		
	V_S = 300.1				
		log dO = 0.8808			
		log q = 0.4898	q = 3.1		

3.—*Q-N*

R_Q = 39.69	V_Q = 262.545	log d = 3.3850	
R_N = 106.50	V_N = 329.304	log d′ = 2.9039	
O = 66.81	C = 66.759	log sin 1′ = 6.1961	
C = 66.759	(V_Q+V_N) = 191.8	colog D = 6.6690	
dO = 5′.1	V_{NQ} = 41.5	$\log \frac{dd' \sin 1'}{D}$ = 9.1540	S = 0.1
	V_S = 150.3		
		log dO = 0.7076	
		log q = 9.8616	q = 0.7

The three loci are now plotted, as shown in Figure 12, and the values of Δx and Δy are measured on the sketch.

x_A = 50 707.0	y_A = 463 785.0	
Δx = + 2.4	Δy = —3.3	*Scale*:—1 cm. = 1.0 m
x_O = 50 709.4	y_O = 463 781.7	

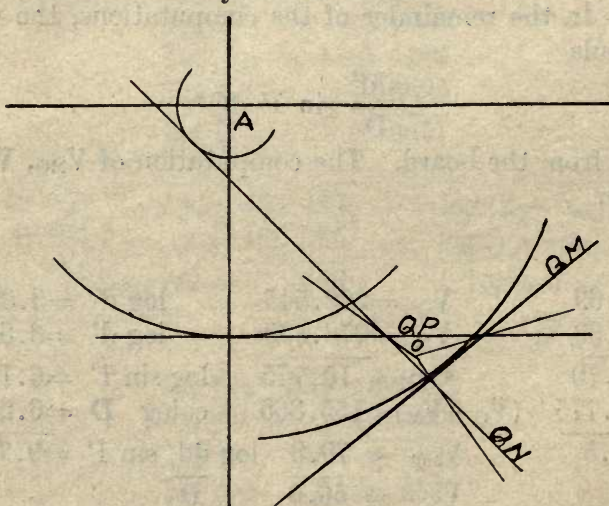


Figure 12.

LOCATION OF A POINT BY RECOUPEMENT (*Resection*)

This procedure is a combination of intersection and the three-point problem. Sights have been taken to known points from the desired point O, which has also been sighted from other known points. The location of the approximate point is graphically obtained, and that of the desired point derived from computations as explained above. Formulas to be used are:

$$q = \frac{dd'}{D} \cdot dO' \cdot \sin 1'.$$

$$a = D \cdot dO' \cdot \sin 1'.$$

$$V_S = V_{AM} + V_{AN} - V_{MN}.$$

The point is chosen in the polygon obtained, after considering the sensitivities of the several loci.

ECCENTRIC STATIONS

The conditions of fieldwork sometimes require the use of eccentric stations, at a known station or at a desired point. Two methods of computation are possible, and either may be followed out.

I.—Reduce the angles to what they would have been if the measurements had been made at the center.

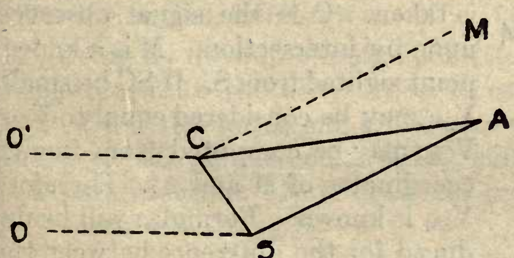


Figure 13.

Let C represent the signal, S the instrument. Let CS, the eccentric distance, equal L. The reading on C is r_0, on A it is r. Through C draw CM parallel to SA. The problem is to compute what the reading would have been if the plate were transported to C, remaining parallel to its original position.

If SO represents the zero at S, CO' parallel to SO represents the zero at C. The reading at C would have been O'CA.

$$\text{Angle } O'CA = O'CM + MCA$$
$$= OSA + MCA$$

But in triangle CSA, CS /CA = sin CAS /sin CSA.

Call CA, the distance to the station whose direction is being reduced, D.

Then, $L/D = \sin CAS / \sin (r - r_0)$.

But angle CAS is equal to angle MCA, or the desired reduction, and we will call it k. Then, $\sin k = \dfrac{L \sin (r - r_0)}{D}$.

If k be expressed in seconds, and is small, $k'' \sin 1'' = \frac{L \sin (r - r_0)}{D}$

$$k'' = \frac{L \sin (r - r_0)}{D \quad \sin 1''}$$

This is the same as the regular U. S. formula for reduction to center.

II.—In the location by recoupement (resection), it may happen that eccentric observations are made at the desired point, but that the observations from the known points are made on the center or the signal.

Incidental to this computation, is the computation of differences in coordinates between center and eccentric station. This operation will be inserted here parenthetically. In the location by recoupement, the *signal* may be plotted by intersection. The approximate coordinates can be found for C, then, the center. This we will call the point "A_1". The problem in hand is to compute the coordinates of a corresponding approximate point A for S, the eccentric station. These two points A and A_1 are only approximate points for S and C, but the differences between the coordinates of A and A_1 are correct and are the actual differences between those of S and C.

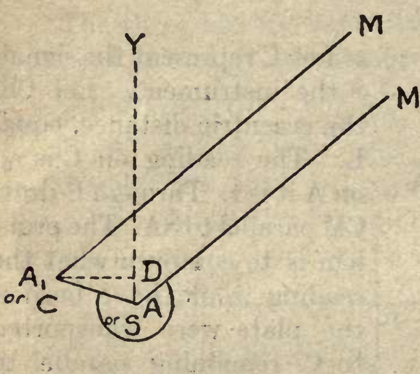


Figure 14.

In the figure, S is the eccentric station where the resection angle is taken. C is the signal observed upon by intersection. M is a known point sighted from S. If SC be small, V_{SM} may be considered equal to V_{CM}. V_{CM} may be computed from known coordinates of M and A_1. Therefore V_{SM} is known. Formulas will be deduced for the difference between the coordinates of S and C, or A_1 and A. Suppose the reading on C to be r_0 and on M to be r.

$$V_{SC} = V_{SM} - (r - r_0).$$

$$\Delta x = CD = SC \sin CSD$$

$$= L \sin V_{SC}$$

$$\Delta y = DS = SC \cos CSD$$

$$= L \cos V_{SC}.$$

Then if we are computing the coordinates of S from those of C,

$$\left.\begin{array}{l} x_S = x_C + \Delta x \\ y_S = y_C + \Delta y \end{array}\right\} \text{ or } \left\{\begin{array}{l} x_a = x_{a1} + \Delta x \\ y_a = y_{a1} + \Delta y \end{array}\right.$$

Now, in the location by recoupement, suppose observations from M and N, known points, to be taken on C, the signal at the desired point. Then suppose with the instrument at S, the eccentric station at the unknown point, the angle is measured between two known stations, as N and P. Now, let us find graphically the approximate point A_1, for the center C, by intersection. By computing S from C as described above, the corresponding approximate point A is found for S, the instrument.

Then the computation is made by *computing loci for S*, as shown in the following:

A locus containing S can be computed from the angle observed on N and P, by the ordinary three-point method of computation, using approximate point A. Another locus containing S, to be computed from the observations taken from M or N on C is made as follows:—*Compute the displacement q, using the point A_1 as the approximate point then draw the locus as if it had been computed for the approximate point A.*

The intersection of the two loci containing S gives the location of S on the sketch. Then Δx and Δy, scaled off and added to the coordinates of A, give x_S and y_S desired.

To show why the procedure discussed above is followed out to obtain the locus containing S from the observation from M on C, the following explanation is given:

Using A_1 the approximate point for C, A_1O_1 is computed as the displacement to the intersection locus, for the angle observed at M, say. OC is this locus, and it contains C. Draw O_1R parallel to A_1A, and drop AO perpendicular to OC, cutting O_1R at R. Then AA_1O_1R is a parallelogram. Now draw RS parallel to OC. This is obviously a locus containing S.

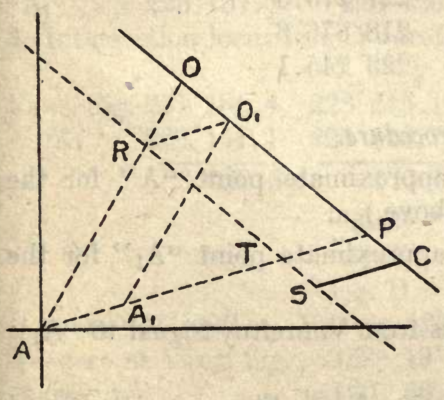


Figure 15.

But $AR = A_1O_1$. Hence the *Locus* RS *containing S* would have been obtained if we had plotted from A as the approximate point the displacement A_1O_1. Then on a large scale sketch, with A plotted and A_1 not plotted, we can draw loci as follows containing S: Any three-point locus, computed as usual with A as the approximate point. Any intersection locus, with observation made on C, the displacement having been computed from A_1 but plotted from A on this sketch.

The desired point S should be chosen from the triangle or polygon of error, by weighting the loci by the ordinary methods for intersection and three-point problems.

This method of treating the eccentric station computation gives as the result the coordinates of S, the eccentric station. Method No. I is usually followed in case a known station is occupied eccentrically.

EXAMPLE OF COMPUTATION OF RECOUPEMENT PROBLEM

Computation of Connontroy Signal

Data

At Connontroy Eccentric Eccentric Distance = 47.0 ft.

Champenoise Church	00°	00′	00″
Cote 209	211	27	09
Vaurefroy Signal	174	22	20
Connontroy Signal	264	28	51

Compute loci between: Champenoise Church—Cote 209
Vaurefroy Signal —Cote 209

At *Vaurefroy Signal* V—zero=152° 19′ 45″
Connontroy *Signal* 120° 56′ 59″

At *Champenoise Signal* V—zero=327° 26′ 26″
Connontroy *Signal* 66° 34′ 04″

Approximate Point for Connontroy Eccentric: X= 228 152
Y = 223 435

Coordinates of Known Points

Point	X	Y
Champenoise Church	225 016.7	223 940.3
Champenoise Signal	226 718.4	221 275.0
Cote 209	233 474.3	218 876.3
Vaurefroy Signal	231 183.4	223 245.1

Method of Procedure.

1. Determine graphically the approximate point "A" for the eccentric of Connontroy. (Given above.)

2. Compute the corresponding approximate point "A_1" for the center, or Connontroy Signal.

3. Compute an intersection locus from Vaurefroy Signal to "A_1" or center.

4. Compute an intersection locus from Champenoise Signal to "A_1".

EXAMPLE OF COMPUTATION OF RECOUPEMENT PROBLEM—*Continued.*

5. Compute a three point locus for "A" using Champenoise Church—Cote 209.

6. Compute a three point locus for "A" using Vaurefroy Signal—Cote 209.

7. Plot all loci using either "A" or "A_1" as approximate point.

1. Approximate point "A" for eccentric=X 228 152
Y 223 435

2. Computation of corresponding approximate point "A_1" for center.

	x	y		
"A"	228 152	223 435	log. Δx.	3.72610
Cote 209	233 474.3	218 876.3	log. Δy.	3.65884
	5 322.3	4 558.7		0.06726=log. tan. v

v=49° 25′ 09″ V_A—Cote 209 =130° 34′ 51″
—R on Cote 209 =211 27 09

Approx. V—zero at A, 279 07 42
R on center 264 28 51

183 36 33 =V_{ECC}—Center (approx.)

Eccentric Distance is 47.0 ft.=D. (reduce to meters.)

Log. D	1.15611	Log. D.	1.15611
Log. sin. V_E—$_C$	8.79899	Log. cos. V_E—$_C$	9.99913
	9.9 510		1.15524
	—x=0.9		—y=14.3

	x	y
"A"	228 152	223 435
	—0.9	—14.3
"A_1"	=228 151.1	223 420.7

3. Intersection locus from Vaurefroy Signal to "A_1".

	x	y		
Vaur. Sig.	231 183.4	223 245.1	log. Δx	3.48177
"A_1"	228 151.1	223 420.7	log. Δy	2.24452
	3 032.3	175.6	"tan v	1.23725

log. Δx	3.48177
"sin v	9.99927
log. D	3.48250

v=86° 41′ 09″ $V_{V—A1}$=273° 18′ 51″, computed y-azimuth
V—zero at Vaur. Sig. =152° 19′ 45″
R on "A_1" =120 56 59
273 16 44 =observed y-azimuth.

EXAMPLE OF COMPUTATION OF RECOUPEMENT PROBLEM—*Continued.*

computed y-az.	273° 18′ 51″		
observed y-az.	273 16 44	log. D	3.48250
dO	00 02 07	″sin 1′	6.46373
	2.1167′	log dO′	0.32566
		log. q	0.27189
			q=1.87 meters

Computed az. greater than observed, therefore locus is on lower part of circle.

4. Intersection locus from Champenoise Signal to "A_1".

	x	y		
Champ. Sig.	226 718.4	221 275.0	log. Δx	3.15616
"A_1"	228 151.1	223 420.7	log. Δy	3.33157
	1 432.7	2 145.7	log. tan. v.	9.82459

log. Δx 3.15616
″sin v 9.74453
log. D 3.41163

v=33° 43′ 53″ V_{CA1} = 33° 43′ 53″ computed y-azimuth
V—zero at Champ. Sig. = 327° 26′ 26″
R on A_1 = 66 34 04

34 00 30 observed y-azimuth.
33 43 53 computed y-azimuth.

dO = 00 16 37 O greater than C.
dO = 16.617

log. D	3.41163
″sin 1′	6.46373
″dO′	1.22055
log. q	1.09591
q	= 12.47 meters

Locus on lower part of circle.

5. Computation of three point locus for "A" using Champenoise Church and Cote 209.

	x	y		
Champ. Ch.	225 016.7	223 940.3	log. Δx	3.49628
"A"	228 152	223 435	log. Δy	2.70355
	3 135.3	505.3	″tan v	0.79273

log. Δx 3.49628
″sin v 9.99443
log D. 3.50185

v=80° 50′ 41″ $V_{A—C}$=279° 09′ 19″

EXAMPLE OF COMPUTATION OF RECOUPEMENT PROBLEM—*Continued.*

	x	y		
Cote 209	233 474.3	218 876.3	log. Δx	3.72610
"A"	228 152	223 435	log. Δy	3.65884
	5 322.3	4 558.7	" tan v.	0.06726

log. Δx	3.72610
" sin v	9.88051
" D	3.84559

v = 49° 25′ 09″ $V_{A—COTE}$ 209 = 130° 34′ 51″

	x	y		
Champ. Ch.	225 016.7	223 940.3	log. Δx	3.92725
Cote 209	233 474.3	218 876.3	log. Δy	3.70449
	8 457.6	5 064.0	" tan v	0.22276

log. Δx	3.92725
" sin v	9.93347
log. D	3.99378

v = 59° 05′ 21″ V_C Ch.—Cote 209 = 120° 54′ 39″

Champ. Ch. R =	00°	00′	00″	V =	27°	09′	19″
—Cote 209 R =	211	27	09	—V =	130	34	51
Observed	148	32	51		148	34	28
Computed	148	34	28				
dO	00	01	37				
dO		1.617′					

log. d	3.50185	
log. d′	3.84558	
colog D	6.00623	
log. S	3.35366	S = 2257.7^m
" sin 1′	6.46373	
log. dO′	0.20871	
log. q	0.02610	q = 1.06^m

V_A—c	279°	09′	19″
+ V_A—c 209	130	34	51
	409	44	10
— V_C—c	120	54	39
	288	49	31

= V_S, direction of locus.

O, observed angle is less than C, the computed angle. Locus on upper part of circle.

6. Computation of three point locus for "A" using Vaurefroy Sig. and Cote 209.

EXAMPLE OF COMPUTATION OF RECOUPEMENT PROBLEM—*Continued.*

Vaur. Sig.	231 183.4	223 245.1	log. Δx 3.48165	log. Δx 3.48165
"A"	228 152	223 435	log. Δy 2.27852	"sin v 9.99915
	3 031.4	189.9	"tan v 1.20313	"D 3.48250

v=86° 24′ 56″ V_A—v. Sig.=93° 35′ 04″

Cote 209 to "A" see computation in "5"

v=49° 25′ 09″ V_A—Cote 209=130° 34′ 51″

log. D=3.84559

Cote 209	233 474.3	218 876.3	log. Δx 3.36001
Vaur. Sig.	231 183.4	223 245.1	log. Δy 3.64036
	2 290.9	4 368.8	"tan v 9.71965

log. Δx	3.36001
"sin v	9.66690
log. D	3.69311

v=27° 40′ 19″ V_C—v=332° 19′ 41″

Cote 209	211°	27′	09″	V=130°	34′	51″
Vaur. Sig.	174	22	20	—V= 93	35	04
observed	37	04	49	36	59	47
computed	36	59	47			
dO	00	05	02			
dO		5.0333′				

log. d	3.48250	
log. d′	3.84559	
colog D	6.30689	
log. S	3.63498	S=4315.0^m
log. sin 1′	6.46373	
log. dO′	0.70185	
log. q	0.80056	q=6.32^m

V+V	=224°	09′	55″	
—V	=332	19	41	
V_S	71	50	14	direction of locus

Observed angle greater than computed angle.
Locus on lower part of circle.

Graph for Problem Below.

No. 1. Intersection locus, Vaurefroy Signal—A_1.
No. 2. Intersection locus, Champenoise Signal—A_1.
No. 3. Three point locus, Champenoise Church—Cote 209—A.
No. 4. Three point locus, Vaurefroy Signal—Cote 209—A.

$\Delta x = +12.6^m$ $\Delta y = -3.0$

	x	y
A=	228 152	223 435
	+12.6	—3.0
Connontroy Ecc. =	228 164.6	223 432.0
A_1=	228 151.1	223 420.7
	+12.6	—3.0
Connontroy Center =	228 163.7	223 417.7

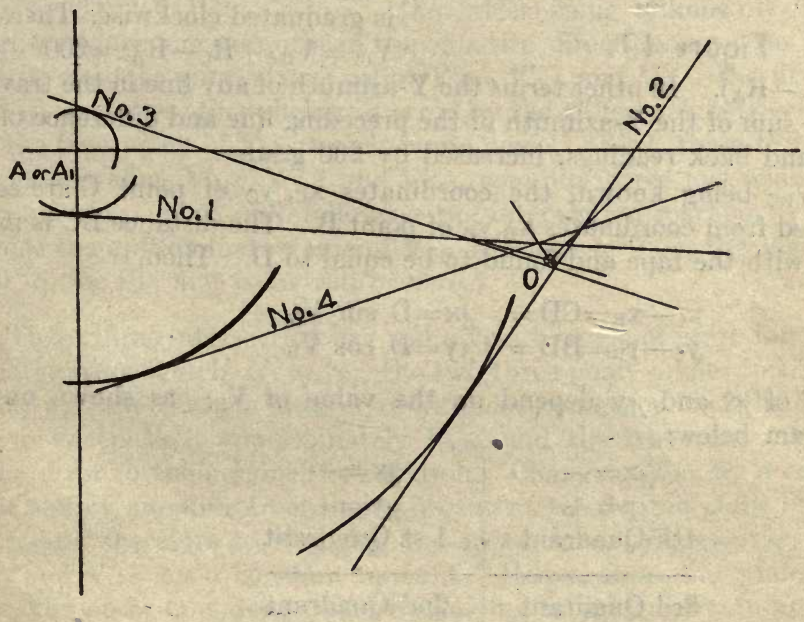


Figure 16.

Scale 1cm.=2m.

LOCATION BY TRAVERSING

From a known station point, A, a traverse is carried on passing through the desired point. The question is to determine the coordinates of the successive vertices of the traverse line, and consequently the coordinates of the desired points. Let A, B and C be the vertices of the traverse. The instrument is set up over A, and a round of angles taken to known points, allowing the observer to compute the coordinates of A if unknown, and determine the V-zero. Point B being included in the sights, the Y-azimuth V_{AB} of line AB is derived from the corrosponding reading R_B.

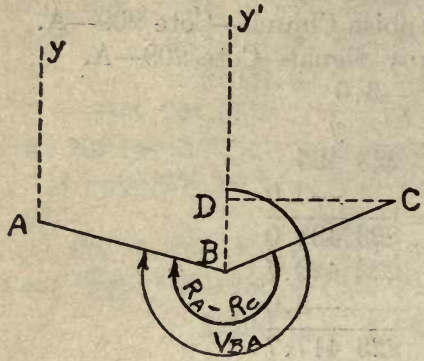


Figure 17.

$$V_{AB} = V\text{-zero} + R_B.$$

Set up the instrument over B and sight successively A and C, the corresponding readings being R_A and R_C. The Y-azimuth of side BC is

V_{BC} = angle $y'BC$ = $y'BA - CBA = V_{BA} - CBA$.

But angle CBA is the difference $R_C - R_A$ if the instrument used is graduated clockwise. Therefore $V_{BC} = V_{BA} + R_C - R_A = 200^G + V_{AB} + (R_C - R_A)$. In other terms the Y-azimuth of any line in the traverse is the sum of the Y-azimuth of the preceding line and difference of the fore and back readings, increased by 200 grades.

V_{BC} being known, the coordinates x_C, y_C of point C are easily derived from coordinates x_B, y_B of point B. The distance BC is measured with the tape and found to be equal to D. Then,

$$x_C - x_B = CD = \Delta x = D \sin V_{BC}$$
$$y_C - y_B = BD = \Delta y = D \cos V_{BC}$$

Signs of Δx and Δy depend on the value of V_{BC}, as shown on the diagram below:

$\Delta x = -$ $\Delta y = +$ 4th Quadrant	$\Delta x = +$ $\Delta y = +$ 1 st Quadrant
3rd Quadrant $\Delta x = -$ $\Delta y = -$	2nd Quadrant $\Delta x = +$ $\Delta y = -$

COMPUTATION OF THREE-POINT OBSERVATIONS BY THE APPROXIMATE ORIENTATION METHOD

Suppose angles are measured by three-point method at point O, with observations on M, N and P. Obtain a point as near O as possible, by one of the graphical methods already given for finding the approximate point in the three-point problem. Call this point A. With its coordinates and the coordinates of one of the known points, say M, we may compute the Y-azimuth of V_{MA} or V_{AM}.

Using the value of V_{AM} as an approximate value, V'_{OM}, for the Y-azimuth from O to M, compute an approximate V-zero at O. This differs from the correct V-zero by the angle OMA. Using this V-zero, compute from the *observed* angles approximate Y-azimuths to N and P, giving V'_{ON} and V'_{OP}. (Note:—It is to be remembered that if Y-azimuths had been *computed* from A to M, N and P, they would not differ by these observed angles. Also notice that each of the three approximate Y-azimuths found as stated above differs from its true value by a constant amount, namely the difference between the Y-azimuths V_{AM} and V_{OM}, the latter being unknown.) From these, we know the azimuths in the opposite directions over the lines. That is, we have *approximate values* V'_{MO}, V'_{NO} and V'_{PO}, the first being V_{MA} and the others being in error by the difference between V_{MA} and the true value of V_{MO}, which is unknown. With these three approximate values V'_{MO}, V'_{NO} and V'_{PO}, compute three loci, using the approximate point A and following the intersection method. (Note: —Since the approximate value of V_{MO} is equal to V_{MA}, the displacement, q, for the first locus will be zero.)

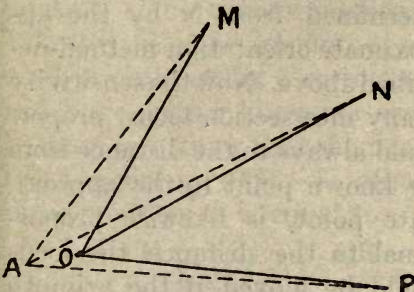


Figure 18.

These three intersection loci will give a triangle of error for O on the large scale sketch. However, the two three-point angles furnish no check; and the triangle of error results *solely* on account of the difference between V_{MA}, approximately V_{MO}, and the true value of V_{MO}, or the error in the assumed orientation. Changing V_{MA} by a certain value simply amounts to changing V-zero at the desired point by this value, and therefore to changing the approximate Y-azimuths V'_{MO}, V'_{NO} and V'_{PO} by a constant amount. Hence, from the triangle of error, the point O is determined by moving each locus by an amount proportional to its sensitivity (or proportional to D), all being moved in directions corresponding to changes in the approximate values V'_{MO}, V'_{NO} and V'_{PO} *in the same direction*, the movement being such as to reduce the triangle of error to a single point. This then will be the true point O determined from the two angles without check.

This method corresponds exactly to the three-point location as determined with a plane-table. The use of an approximate V-zero corresponds to the approximate orientation of the plane-table, and the selection of the final point in the triangle of error is performed in the same manner as on the plane-table.

Let us now consider how we would use this method in connection with ordinary intersection observations,—that is, in recoupement, or resection. Let Figure 19 represent a portion of the large scale adjustment sketch. BM is the locus determined from M, and BN is the locus determined from N by the approximate orientation method described above. Now the sensitivity of any intersection locus, proportional always to the distance from the known point to the approximate point, is likewise proportional to the distance the locus would be moved if the azimuth of the observed line were changed by a certain amount. Suppose we change our original orientation by a certain arbitrary amount, thereby changing V'_{MO}, V'_{NO} and V'_{PO} by the same amount. Then locus BM is transposed parallel to itself to some position like CD, and BN to CE, the distances CK and CH by which the two loci are transposed being proportional, respectively, to the relative sensitivities or to D_{AM} and D_{AN}. Draw BC.

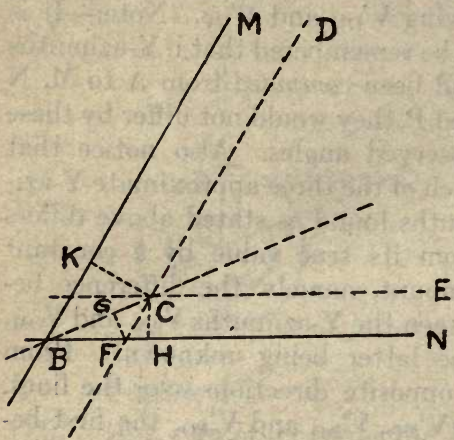


Figure 19.

Now, in changing the original orientation, we have changed the Y-azimuths of both of the loci by the *same* amount, and hence have *not* changed the angle between them. Therefore BC is the locus of all the points at which the same angle would be observed between the two known points M and N. In other words, BC is the *three-point locus* for the angle observed at O between M and N. The sensitivity of this three-point locus is found as follows:—

Let F be the point at which CD intersects BN. Draw FG perpendicular to BC. Now F represents the point to which B would be moved if we changed the observed Y-azimuth for the intersection locus BM without changing the Y azimuth for BN. This amounts to changing the value of the original angle observed between the two known points M and N by the same amount by which we have changed the observed Y-azimuth for the locus, BM. Therefore F is a point through which the *three-point locus*, BC, would pass if the observed angle were changed by the given amount, and FG represents the *distance* by which the three-point locus would be transposed. Therefore FG represents the *sensitivity* of the three-point locus, and is measured on the same scale as CK and CH.

For example, suppose $D_{MA}=7000$, and $D_{NA}=6000$. Suppose we make KC equal to some arbitrary distance like 14mm., which is convenient. Then HC must be made equal to 12 mm. in order that BM and BN may be transposed by distances proportional to their sensitivities. These transpositions would result by changing V'_{MO} and V'_{NO} by the same amount, or in other words by changing the approximate orientation by a certain amount, this amount being something which we neither know nor wish to know. Then if we find FG equal to 8mm., we know that the relative sensitivity of the three-point locus, BC, is equal to 4000. In other words, the relative sensitivity of the three-point locus is equal to

$$\frac{CH}{D_{NA}} \times GF, \text{ or to } \frac{CK}{D_{MA}} \times GF.$$

We can now use our three-point locus in connection with any intersection locus. A locus determined by an intersection observation from P, for instance, would be plotted, and its relative sensitivity would be taken as D_{PA}. And the three-point locus would be combined with the intersection locus, as in the recoupement method.

Remark: It is of greatest importance that each locus be plotted on the proper side of its circle of radius "q", consistent with the computations. In the case of the approximate orientation problem the following rule can be followed:

When "dO" is positive, the line of sight is plotted to the right of the approximate point as one faces the known triangulation station. When "dO" is negative, the line of sight is plotted to the left.

EXAMPLE OF COMPUTATION.

Data

From the desired point O, sights have been taken and readings noted as follows:

Sight on

M	20°	30′	40″	54 895	463 060	Coordinates of Approximate Point A.
P	89	55	05	51 507	459 580	X= 50 704
Q	156	58	47	48 688	462 438	Y=463 781
N	217	03	47	49 989	464 141	

Computations

M.

$X_M = 54\ 895$	$Y_M = 463\ 060$	$\log (X_M - X_A) = 3.62232$
$X_A = 50\ 704$	$Y_A = 463\ 781$	$\log (Y_M - Y_A) = 2.85794$
$(X_M - X_A) = 4\ 191$	$(Y_M - Y_A) = 721$	$\log \tan v_M = 0.76438$
		$v_M = 80^\circ\ 14'\ 19''$
		$V_{A-M} = 99^\circ\ 45'\ 41''$

EXAMPLE OF COMPUTATION—*Continued*

V_{A-M}	$=99^\circ\ 45'\ 41''$	$\log (X_M - X_A)$	$=3.62232$
$-R_M$	$=20\ \ 30\ \ 40$	$\log \sin v_M$	$=9.99367$
V_Z app.	$=79\ \ 15\ \ 01$	$\log D$	$=3.62865$
$-V_Z$ mean	$=79\ \ 22\ \ 50$		
dO	$= -07\ \ 49$	$\log (Y_M - Y_A)$	$=2.85794$
dO'	$= -7.817'$	$\log \cos v_M$	$=9.22929$
		$\log D$	$=3.62865$

$\log \sin 1' = 6.46373$
$\log D = 3.62865$
$\log S = 0.09238$ S, sensitivity$=1.237$
$\log dO' = 0.89304$
$\log q = 0.98542$ $q=9.67^m$. Locus plotted on upper right hand side of circle.

N.

X_N	$=49\ 989$	Y_N	$=464\ 141$	$\log (X_N - X_A)$	$=2.85431$
X_A	$=50\ 704$	Y_A	$=463\ 781$	$\log (Y_N - Y_A)$	$=2.55630$
$(X_N - X_A)$	$= 715$	$(Y_N - Y_A)$	$= 360$	$\log \tan v_N$	$=0.29801$
				v_N	$= 63^\circ\ 16'\ 30''$
				V_{A-N}	$=296\ \ 43\ \ 30$

V_{A-N}	$=296^\circ\ 43'\ 30''$	$\log (X_N - X_A)$	$=2.85431$
$-R_N$	$=217\ \ 03\ \ 47$	$\log \sin v_N$	$=9.95094$
V_Z app.	$= 79\ \ 39\ \ 43$	$\log D$	$=2.90337$
$-V_Z$ mean	$= 79\ \ 22\ \ 50$	$\log (Y_N - Y_A)$	$=2.55630$
dO	$= +16\ \ 53$	$\log \cos v_N$	$=9.65293$
dO'	$= +16.883'$	$\log D$	$=2.90337$

$\log \sin 1' = 6.46373$
$\log D = 2.90337$.
$\log S = 9.36710$ S, sensitivity$=0.233$
$\log dO' = 1.22745$
$\log q = 0.59455$ $q=3.93^m$

Locus plotted on right hand side of circle.

P.

X_P	$=51\ 507$	Y_P	$=459\ 580$	$\log (X_P - X_A)$	$=2.90472$
X_A	$=50\ 704$	Y_A	$=463\ 781$	$\log (Y_P - Y_A)$	$=3.62335$
$(X_P - X_A)$	$= 803$	$(Y_P - Y_A)$	$=4\ 201$	$\log \tan v_P$	$=9.28137$
				v	$= 10^\circ\ 49'\ 17''$
				V_{A-P}	$=169^\circ\ 10'\ 43''$

EXAMPLE OF COMPUTATION—*Continued*

$V_{A—P}$	$= 169^o\ 10'\ 43''$	$\log (X_P — X_A)$	$= 2.90472$
$—R_P$	$= 89\ \ 55\ \ 05$	$\log \sin v_P$	$= 9.27358$
V_Z app.	$= 79\ \ 15\ \ 38$	$\log D$	$= 3.63114$
$—V_Z$ mean	$= 79\ \ 22\ \ 50$		
dO	$= —07\ \ 12$	$\log (Y_P — Y_A)$	$= 3.62335$
dO'	$= —7.200'$	$\log \cos v_P$	$= 0.99221$
		$\log D$	$= 3.63114$
$\log \sin 1'$	$= 6.46373$		
$\log D$	$= 3.63114$		
$\log S$	$= 0.09487$	S, sensitivity $= 1.240$	
$\log dO'$	$= 0.85733$		
$\log q$	$= 0.95220$	$q = 8.96^m$.	

Locus plotted on right hand side of circle.

Q. $X_Q = 48\ 688$ $\quad Y = 462\ 438 \quad \log (X_Q — X_A) = 3.30449$

$X_A = 50\ 704 \quad Y = 463\ 781 \quad \log (Y_Q — Y_A) = 3.12808$

$(X_Q — X_A) = 2\ 016 \quad (Y_P — Y_A) = 1\ 343 \quad \log \tan v_Q = 0.17641$

$v_Q = 56^o\ 19'\ 45''$

$V_{A—Q} = 236^o\ 19'\ 45''$

$V_{A—Q}$	$= 236^o\ 19'\ 45''$	$\log (X_Q — X_A)$	$= 3.30449$
$—R_P$	$= 156\ \ 58\ \ 47$	$\log \sin v_Q$	$= 9.92025$
V_Z app.	$= 79\ \ 20\ \ 58$	$\log D$	$= 3.38424$
V_Z mean	$= 79\ \ 22\ \ 50$		
dO	$= —01\ \ 52$	$\log (Y_Q — Y_A)$	$= 3.12808$
dO'	$= —1.867'$	$\log \cos v$	$= 9.74384$
		$\log D$	$= 3.38424$
$\log \sin 1'$	$= 6.46373$		
$\log D$	$= 3.38424$		
$\log S$	$= 9.84797$	S, sensitivity $= 0.705$	
$\log dO'$	$= 0.27114$		
$\log q$	$= 0.11911$	$q = 1.32^m$.	

Locus plotted on lower side of circle

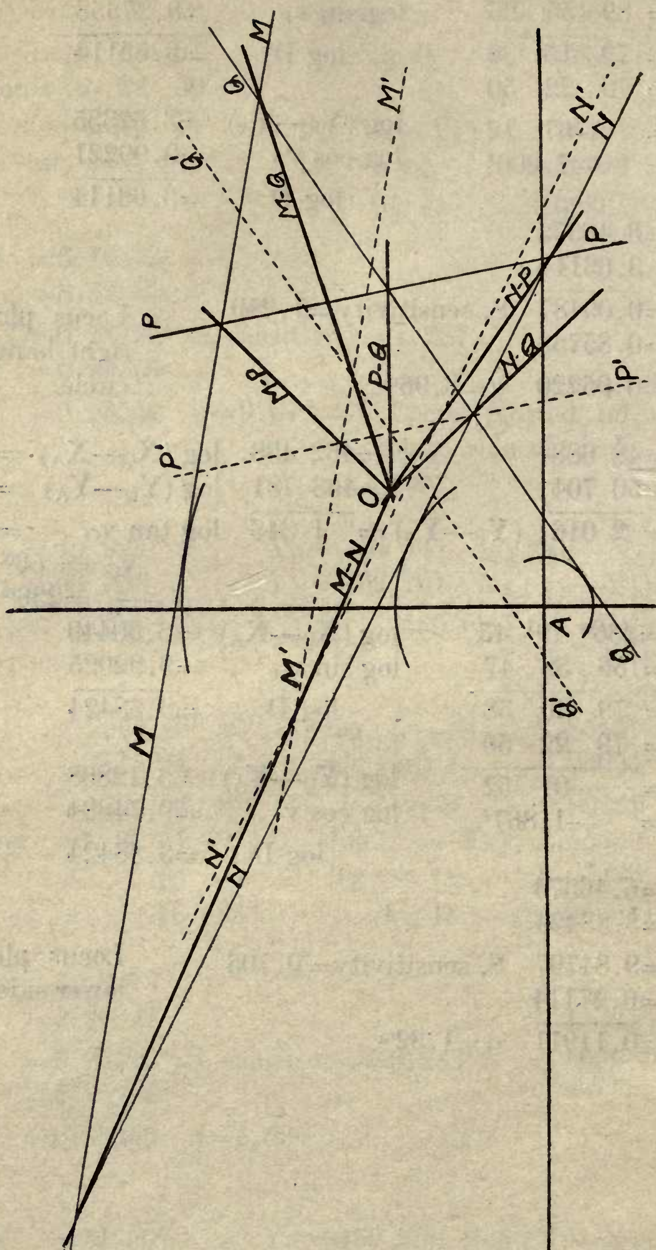


Figure 20.

RADIUS METHOD FOR SOLUTION OF THREE-POINT PROBLEM

The following method was devised as a simplification of the computation for the Three-Point Problem, as previously developed in this paper.

Suppose M and N to be two known points sighted, and let us suppose that we wish to plot the locus which contains the desired point O, by virtue of the observed angle MON. Suppose the approximate point A has been graphically determined. Then the coordinates of M, N and A are all known; call them x_M, y_M and x_N, y_N and x_A, y_A. In the figure, draw MP perpendicular to MN, and draw NP making angle MNP equal to the complement of the observed angle MON. Then the point P where these lines intersect lies on the circle MNO.

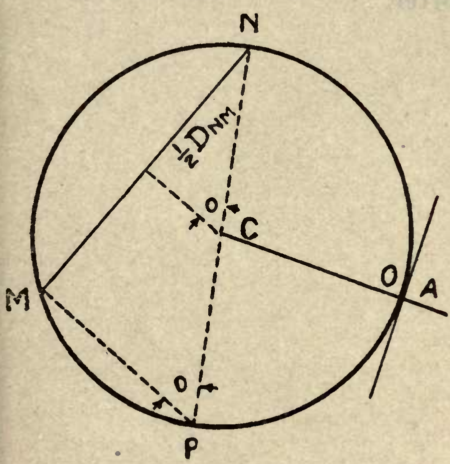


Figure 21.

In case the observed angle exceeds 90°. replace O by (180°— O); or (90°—O) by (90°+O).

From the coordinates of M and N, let us compute the distance D_{MN} from N to M, and the Y-azimuth V_{NM} of the line NM. Let us designate by O the angle observed between M and N. If we subtract the complement of O from V_{NM} we obviously obtain the Y-azimuth V_{NC} of the line NC. Then calling r the radius of the circle MNO, we have

$$r = \frac{D_{NM}/2}{\sin O}$$

and the differences in coordinates between N and C are

$$\Delta x = r \sin V_{NC}.$$
$$\Delta y = r \cos V_{NC}.$$

These values added to or subtracted from the coordinates of N give those of C, the center of the circle. Call them X_C and Y_C.

Next let us compute the Y-azimuth and distance from the approximate point A to C. Now when we come to plot on our large scale adjustment sketch the locus containing O determined by measuring the angle MON, the distance from A to this locus is easily seen to be $r—D_{AC}$ and the direction of the locus, or the direction of the circle at

RADIUS METHOD FOR SOLUTION OF THREE-POINT PROBLEM—*Continued.*

that point, is $V_{AC}+90^o$. The locus of course becomes a straight line on the adjustment sketch. The proper side of A upon which to plot the locus is shown by the algebraic sign of $r—D_{AC}$.

The advantages in the method described above are that it is much easier to understand than any of those used heretofore, and that the actual computation is somewhat shorter.

BAUDE'S METHOD OF SOLUTION OF THREE POINT PROBLEM—Continued.

that point, is V [illegible] The locus of [illegible] becomes a straight line on the adjustment sketch. The prolongation of A [illegible] which [illegible] the locus is [illegible]

[illegible]